Reši naslednje nepopolne diferencialne enačbe drugega reda:

1. $x^2 y'' = y'^2$

2. $y'' = 2yy'$

3. $\left(\dfrac{d^2 y}{dx^2}\right)^2 + \dfrac{dy}{dx} = x\dfrac{d^2 y}{dx^2}$

4. $y'^2 + 2yy'' = 0$

5. $y'' = e^y$.

Izračunaj partikularne rešitve naslednjih diferencialnih enačb:

6. $y'' = 1$, $y(0) = -2$ *in* $y'(0) = 0$

7. $y'' = \sin x$, $y(0) = 0$ in $y'(0) = 1$

8. $\dfrac{d^2 y}{dx^2} = e^x$, $y(-\infty) = 0$ in $y'(-\infty) = 0$

9. $yy'' = 2y'^2$, $y(0) = 4$ in $y'(0) = 0$

Izračunaj splošno rešitev naslednjih homogenih diferencialnih enačb drugega reda s konstantnimi koeficienti:

11. $y'' - 2y + 10y = 0$

12. $y'' + y' - 2y = 0$

13. $y'' - 6y' + 25y = 0$

14. $9\dfrac{d^2 y}{dx^2} - 12\dfrac{dy}{dx} + 4y = 0$

15. $y'' - 4y' + 5y = 0$.

Določi partikularno rešitev homogenih diferencialnih enačb drugega reda s konstantnimi koeficienti, ki zadoščajo podanim začetnim pogojem:

16. $y'' + y = 0$, $y\left(\dfrac{\pi}{2}\right) = 0$ in $y'\left(\dfrac{\pi}{2}\right) = -2$

17. $4y'' - 8y' + 5y = 0$, $y(0) = 0$ in $y'(0) = \dfrac{1}{2}$

18. $\dfrac{d^2 y}{dx^2} + 4\dfrac{dy}{dx} + 4y = 0$, $y(0) = 1$ in $y'(0) = 3$

19. $y'' - 2y' + y = 0$, $y(1) = 0$ in $y'(1) = e$

20. $y'' + 2y' + 2y = 0$, $y(0) = y'(0) = 0$.

Z metodo variacije konstant izračunaj splošno rešitev nehomogenih diferencialnih enačb drugega reda s konstantnimi koeficienti:

21. $y'' - 2y' + y = \dfrac{e^x}{x}$

22. $y'' + 4y = 2\tan x$

23. $y'' + y = \dfrac{1}{\cos x}$

24. $y'' + y = \dfrac{1}{\sin x}$

25. $y'' + 3y' + 2y = \dfrac{1}{1 + e^x}$.

26. $y'' - 2y' - 3y = e^{4x}$

27. $y'' + y = x\sin x$

28. $y'' + 4y' + 4y = xe^{2x}$

29. $y'' + 4y' + 4y = e^{-2x}\ln x$.

Izračunaj splošno rešitev nehomogenih diferencialnih enačb drugega reda s konstantnimi koeficienti z metodo nedoločenih koeficientov:

30. $y'' - y = 2e^x - x^2$

31. $y'' + y = 4\sin x$

32. $y'' + y' - 2y = 4xe^x$

33. $y'' + y = x\sin x$

34. $y'' - 2y' + y = 6xe^{-x}$

35. $y'' + 9y = 4\sin 3x + 2x^2$

36. $y'' + y = x^2 2x + e^{3x}$

37. $y'' + 4y' + 13y = \cos x + \sin x$

38. $y'' + 4y' - 3y = x + 2$

39. $y'' + 5y' + 4y = e^x \sin x$.

Izračunaj splošno rešitev Eulerjevih diferencialnih enačb drugega reda:

40. $x^2 y'' - 4xy' + 6y = 0$

41. $x^2 y'' - xy' - 3y = 0$

42. $x^2 y'' + 5xy' + 5y = 0$

43. $x^3 y'' - 2xy = 6\ln x$

44. $x^2 y'' + 9y' + 25y = 2\cos\ln x$

45. $x^2 y'' - 3xy' + 5y = 3x^2$

46. $x^2 y'' - 6y = 5x^3 + 8x^2$

47. $x^2 y'' - 3xy' + 4y = x + x^2 \ln x$

48. $x^2 y'' - xy' + 4y' = x\sin\ln x$

49. $x^2 y'' - 2y = \sin\ln x$.

Poišči rešitev diferencialnih enačb, ki zadoščajo danim robnim pogojem:

50. $y'' - y = 2x$, $y(0) = 0$ in $y(1) = -1$

51. $x^2 y'' - 6y = 0$, $y(0)$ omejena, $y(1) = 2$

52. $x^2 y'' + 5xy' + 3y = 0$, $y'(1) = 3$ in $y(\infty) = 0$

53. $y'' + y = 1$, $y(0) = 0$ in $y(\pi) = 0$

54. $y'' - 4y = 0$, $y(-2) = y(2) = \text{sh}\,4$

55. $y'' - y' - 2y = 0$, $y(0) = 3$ in $y(1) = 2e^2 + \dfrac{1}{e}$

56. $y'' - y' - 2y = 0$, $y'(0) = 2$ in $y'(\infty) = 0$

57. $x^2 y'' + xy' + 4y = x$, $y(-1) = 2$ in $y(1) = 0$

58. $y'' - \dfrac{y'}{x} - \dfrac{y}{x^2} = \sin x$, $y(1) = 1$ in $y'(2) = 0$

59. $y'' - 2y' = 2e^x$, $y(1) = -1$ in $y'(1) = 0$

60. $y'' - 2y' + y = e^{-x}\sin x$, $y(0) + y'(0) = 0$ in $y(\pi) + y'(\pi) = 1$.

Izračunaj splošno rešitev diferencialnih enačb s konstantnimi koeficienti:

61. $y''' - y'' + 4y' - 4y = 0$

62. $y^{(4)} + 5y'' + 4y = \cos 2x$

63. $y''' - y = \operatorname{ch} x$

64. $y''' - 3y' - 2y = 9e^{2x}$

65. $y''' - 8y = 0$

66. $y^{(5)} - 6y^{(4)} + 9y''' = 0$

67. $y''' - y'' - y' + y = 0$

68. $y^{(4)} + 4y'' + 3y = 0$

69. $y^{(4)} + 32y' + 48y = xe^{-2x}$

70. $y''' - 3y'' + 4y' - 2y = \cos x$

71. $y^{(4)} - 5y'' + 4y = 0$

72. $y^{(5)} + 8y''' + 16y' = 0$

73. $y''' - y'' - y' + y = 0$

74. $y^{(5)} - 10y''' + 9y' = 0$.

Reši naslednje Eulerjeve diferencialne enačbe:

75. $x^3 y''' + xy' - y = 0$

76.

$x^4 y^{(4)} + 6x^3 y''' + 15x^2 y'' + 9xy - 9 = 0$

77. $x^3 y''' + 4x^2 y'' - 8xy + 8y = 0$

78.

$x^3 y''' + 3x^2 y'' + xy' + 8y = 13\cos(\ln x)$.

Rešitve

1. $y = \dfrac{x}{C_1} - \dfrac{1}{C_1^2}\ln(1 + C_1 x) + C_2$

2. $x = C_1 \arctan(C_1 y) - C_2$

3.

$y = \dfrac{x^3}{12} + C_1, \quad y = \dfrac{C_1}{2}x^2 - C_1^2 x + C_2$

4. $x = \dfrac{2}{3C_1}\sqrt{y^3} - C_2$

5.

$x = \pm\dfrac{1}{\sqrt{C_1}}\ln\left[-1 - e^{-y}\left(C_1 + \sqrt{C_1(2e^y + C_1)}\right)\right] - C_2$

6. $y = \dfrac{x^2}{2} - 2$

7. $y = -\sin x + 2x$

8. $y = e^x$

9. $y = 4\operatorname{ch}\sqrt{2}x$

11. $y = e^x(C_1 \sin 3x + C_2 \cos 3x)$

12. $y = C_1 e^x + C_2 e^{-2x}$

13. $y = e^{3x}(C_1 \sin 4x + C_2 \cos 4x)$

14. $y = e^{\frac{2x}{3}}(C_1 + C_2 x)$

15. $y = e^{2x}(C_1 \sin x + C_2 \cos x)$

16. $y = 2\cos x$

17. $y = e^x \sin \dfrac{x}{2}$

18. $y = e^{-\frac{x}{2}}\left(\dfrac{7\sqrt{3}}{3}\sin\dfrac{\sqrt{3}x}{2} + \cos\dfrac{\sqrt{3}x}{2}\right)$

19. $y = e^x(x - 1)$

20. $y = 0$

21. $y = e^x x \ln|x| + e^x(C_1 + C_2 x)$

22. $y = \sin 2x\left(\ln\cos x - \dfrac{1}{2}\right) - x\cos 2x + C_1 \sin 2x + C_2 \cos 2x$

23. $y = x\sin x + \cos x \ln \cos x + C_1 \sin x + C_2 \cos x$

24. $y = -x\cos x + \sin x \ln \sin x + C_1 \sin x + C_2 \cos x$

25. $y = \left(e^{-x} + e^{-2x}\right)\ln\left(1 + e^x\right) + C_1 e^{-x} + C_2 e^{-2x}$

26. $y = C_1 e^{-x} + C_2 e^{3x} + \dfrac{1}{5}e^{4x}$

27. $y = \left(C_1 - \dfrac{x^2}{4}\right)\cos x + \left(C_2 + \dfrac{x}{4}\right)\sin x$

28. $y = \left(C_1 + xC_2\right)e^{-2x} + \left(\dfrac{x}{16} - \dfrac{1}{32}\right)e^{2x}$

29. $y = e^{-2x}\left(C_1 + xC_2\right) + \dfrac{x^2 e^{-2x}}{4}\left(2\ln x - 3\right)$

30. $y = 2 - x^2 + 2\cos x + e^x + C_1 \sin x + C_2 \cos x$

31. $y = -2x\cos x + C_1 \sin x + C_2 \cos x$

32. $y = \left(x - \dfrac{3}{4}\right)e^x + e^{-\frac{x}{2}}\left(C_1 \sin \dfrac{\sqrt{7}x}{2} + C_2 \cos \dfrac{\sqrt{7}x}{2}\right)$

33. $y = \dfrac{1}{4}\left[x\sin x + \left(1 - x^2\right)\cos x\right] + C_1 \sin x + C_2 \cos x$

34. $y = \dfrac{3}{2}(1 + x)e^{-x} + 2\left(C_1 + C_2 x\right)e^x$

35. $y = \dfrac{2}{3}\left(\dfrac{x^2}{3} - x\cos 3x - \dfrac{2}{27}\right) + C_1 \sin 3x + C_2 \cos 3x$

36. $y = 2x^3 - 12x - 12\sin x + \dfrac{e^{3x}}{10} + C_1 \sin x + C_2 \cos x$

37. $y = \dfrac{1}{20}\left(2\sin x + \cos x\right) + e^{-2x}\left(C_1 \sin 3x + C_2 \cos 3x\right)$

38. $y = -\dfrac{3x + 10}{9} + C_1 e^{\left(\sqrt{7}-2\right)x} + C_2 e^{-\left(\sqrt{7}+2\right)x}$

39. $y = \dfrac{e^x}{130}\left(9\sin x - 7\cos x\right) + C_1 e^{-x} + C_2 e^{-4x}$

40. $y = C_1 x^2 + C_2 x^3$

41. $y = \dfrac{C_1}{x} + C_2 x^3$

42. $y = \dfrac{C_1 \sin \ln x + C_2 \cos \ln x}{x^2}$

43. $y = -\dfrac{\ln^2 x}{x} - \dfrac{2\ln x}{3x} + \dfrac{C_1}{x} + C_2 x^2$

44. $y = \dfrac{1}{40}\left(\sin \ln x + 3\cos \ln x\right) + \dfrac{C_1 \sin\left(3\ln x\right) + C_2 \cos\left(3\ln x\right)}{x^4}$

45. $y = 3x^2 - x^2\left(C_1 \sin \ln x - C_2 \cos \ln x\right)$

46. $\quad y = -\dfrac{x^3}{5} - 2x^2 + \ln x + \dfrac{C_1}{x^2} + C_2 x^3$

47. $\quad y = \dfrac{x^2}{6} \ln^3 x + x + x^2 \left(C_1 + C_2 \ln x \right)$

48. $\quad y = \dfrac{x}{2} \sin \ln x - x \left[C_1 \sin \left(\sqrt{3} \ln x \right) - C_2 \cos \left(\sqrt{3} \ln x \right) \right]$

49. $\quad y = C_1 x^2 + \dfrac{C_2}{x} + 0.1 \cos \ln x - 0.3 \sin \ln x$

50. $\quad y = -2x + \dfrac{\operatorname{sh} x}{\operatorname{ch} 1}$

51. $\quad y = \dfrac{2}{x^2}$

52. $\quad y = -\dfrac{3}{x}$

53. $\quad y = 1 - \cos x$

54. $\quad y = 0$

55. $\quad y = 2\dfrac{e^3 - 1}{e^2 - 1}\left(e^{2x} - e^{-x} \right)$

56. $\quad y = -2e^{-x}$

57. $\quad$ ni rešitve

58. $\quad y = \dfrac{\sin x}{2} + \left(1 - \dfrac{x}{2} \right) \cos x$

59. $\quad y = e^{2x-1} - 2e^x + e - 1$

60. $\qquad y = \dfrac{3}{25} e^{-x} \sin x + \dfrac{4}{25} e^{-x} \cos x + \dfrac{21}{25} e^x - \dfrac{21 + e^{-2\pi}}{25} x e^x$

61. $\quad y = C_1 e^x + C_2 \sin 2x + C_3 \cos 2x$

62. $\quad y = -\dfrac{5}{18} \cos 2x - \dfrac{1}{12} \sin 2x + C_1 \sin 2x + C_2 \cos 2x + C_3 \sin x + C_4 \cos x$

63. $\quad y = \dfrac{xe^x}{6} - \dfrac{\operatorname{ch} x}{2} + C_1 e^x + e^{-\frac{x}{2}}\left(C_2 \sin \dfrac{\sqrt{3}x}{2} + C_3 \cos \dfrac{\sqrt{3}x}{2} \right)$

64. $\quad y = \dfrac{9}{4} e^{2x} + C_1 e^{-2x} + C_2 e^x + C_3 x e^x$

65. $\quad y = C_1 e^{2x} + e^{-x}\left(C_2 \cos \sqrt{3}x + C_3 \sin \sqrt{3}x \right)$

66. $\quad y = C_1 + C_2 x + C_3 x^2 + C_4 e^{3x} + C_5 x e^{3x}$

67. $\quad y = C_1 e^{-x} + C_2 e^x + C_3 x e^x$

68. $\quad y = C_1 \sin x + C_2 \cos x + C_3 \sin \sqrt{3}x + C_4 \cos \sqrt{3}x$

69. $\quad y = -\dfrac{1}{72} + \dfrac{x}{48} + e^{2x}\left(C_1 \sin 2\sqrt{2}x + C_2 \cos 2\sqrt{2}x \right) + e^{-2x}\left(C_3 + C_4 x \right)$

70. $\quad y = \dfrac{\cos x + 3 \sin x}{10} + e^x \left(C_1 + C_2 \sin x + C_3 \cos x \right)$

71. $\quad y = C_1 e^x + C_2 e^{-x} + C_3 e^{2x} + C_4 e^{-2x}$

72. $\quad y = C_1 + \left(C_2 + xC_3 \right)\cos 2x + \left(C_4 + xC_5 \right)\sin 2x$

73. $\quad y = \left(C_1 + xC_2 \right)e^x + C_3 e^{-x}$

74. $\quad y = C_1 + C_2 e^x + C_3 e^{-x} + C_4 e^{3x} + C_5 e^{-3x}$

75. $\quad y = C_1 x + \left(C_2 + C_3 \ln x \right) x \ln x$

76. $\quad y = \dfrac{9}{64}\left(4\ln^2 x - 1 \right) + C_1 + C_2 \ln x + C_3 \sin\left(2\sqrt{2}\ln x \right) + C_4 \cos\left(2\sqrt{2}\ln x \right)$

77. $\quad y = \dfrac{C_1}{x^4} + C_2 x + C_3 x^2$

78. $\quad y = \dfrac{8\cos\ln x - \sin\ln x}{5} + \dfrac{C_1}{x^2} + x\left(C_2 \sin\left(\sqrt{3}\ln x \right) + C_3 \cos\left(\sqrt{3}\ln x \right) \right)$