Reši naslednje diferencialne enačbe z ločljivima spremenljivkama:

1. $\left(xy^2+x\right)dx+\left(y-x^2y\right)dy=0$
2. $\left(1+y^2\right)x\,dx+\left(1+x^2\right)dy=0$
3. $\left(1+2y\right)x\,dx+\left(1+x^2\right)dy=0$
4. $xy\left(1+x^2\right)y'=1+y^2$
5. $e^x\left(\dfrac{dy}{dx}+1\right)=1$
6. $y\dfrac{dy}{dx}+x=1$
7. $xy'+y=y^2$
8. $\left(1+y^2\right)dx-xy\,dy=0\ ,\ y(2)=1$
9. $\sin x\ y'=y\ln y\ ,\ y\left(\dfrac{\pi}{2}\right)=1$
10. $\left(2x+1\right)dy+y^2\,dx=0\ ,\ y(4)=1$
11. $2\sqrt{y}\,dx=dy\ ,\ y(0)=1$

Reši naslednje homogene enačbe:

12. $y^2\,dx+x\left(x-y\right)dy=0$
13. $\left(x^3+y^3\right)dx-3xy^2dy=0$
14. $xy\,dx+\left(x^2+y^2\right)dy=0$
15. $\left(x+y\right)dx+\left(y-x\right)dy=0$
16. $xy'-y=y\ln\dfrac{y}{x}$
17. $x^2-y^2+2xyy'=0$
18. $\left(y+\sqrt{x^2-y^2}\right)dx-x\,dy=0$
19. $x\left(\ln x-\ln y\right)dy-y\,dx=0$
20. $y'=\dfrac{y}{x}+e^{\frac{y}{x}}$

Reši naslednje linearne diferencialne enačbe:

21. $y'+y\cos x=\cos x\sin x$
22. $y'-y=e^x$
23. $\dfrac{dy}{dx}=x+y$
24. $y'+ay=ce^{bx}$
25. $y'+x^2y=x^2$
26. $y'-\dfrac{n}{x+1}y=e^x\left(x+1\right)^n$
27. $x\left(x-1\right)y'+\left(1-2x\right)y+x^2=0$

28. $xy' + y = \ln x$

29. $y' - \frac{2}{x} y = x^4$

30. $\left(1 + x^2\right) y' + xy = \frac{1}{1 + x^2}$

31. $y' + y \cos x = e^{-\sin x}$

32. $y' + y \tan x = \sin 2x$

33. $x \frac{dy}{dx} = \left(x - 1\right) e^x + y$

34. $y' = a \sin x + by$

35. $\frac{dy}{dx} + \frac{4xy}{x^2 + 1} = \frac{1}{x^2 + 1}$

Reši naslednje Bernullijeve enačbe:

36. $y' + 2ax^3 y^3 + 2xy = 0$

37. $y' + y = xy^3$

38. $xy' + xy^2 - y = 0$

39. $y' + y = x\sqrt{y}$

40. $\left(x^2 - 1\right) y' - y\left(y - x\right) = 0$

41. $2xyy' - y^2 + ax = 0$

42. $y^2 + \left(1 - xy\right) y' = 0$

43. $y' + \frac{2y}{x} = \frac{2\sqrt{y}}{\cos^2 x}$

44. $x \frac{dy}{dx} + 2y = x^5 y^2$.

Rešitve

1. $y^2 = C\left(x^2 - 1\right) - 1$

2. $\arctan y + \frac{1}{2} \ln\left(1 + x^2\right) = C$

3. $y = \frac{2C - x^2}{2\left(1 + x^2\right)}$

4. $\frac{x}{\sqrt{1 + y^2}\sqrt{1 + x^2}} = C$

5. $y = -x - e^x + C$

6. $y^2 + x^2 - 2x = C$

7. $y = 1 + Ce^x, \quad y = 0$

8. $y = \pm\sqrt{\frac{x^2}{2} - 1}$

9. $\ln y = \frac{1 - \cos x}{\sin x}$

10. $y = \frac{1}{1 + \frac{1}{2} \ln \frac{1 + 2x}{9}}$

11. $y=(1+x)^2$

12. $x=\dfrac{y}{\ln(Cy)}$

13. $x^3-2y^3=Cx$

14. $x^2y^2+\dfrac{y^4}{2}=C$

15. $\sqrt{x^2+y^2}=C\arctan\left(\dfrac{y}{x}\right)$

16. $x=C\ln\dfrac{y}{x}$

17. $y^2+x^2=Cx$

18. $x=Ce^{\arcsin\frac{y}{x}}$

19. $x=ye^{Cy+1}$

20. $x=Ce^{-e^{-\frac{y}{x}}}$

21. $y=\sin x-1+Ce^{-\sin x}$

22. $y=\dfrac{e^x}{2}+Ce^{-x}$

23. $y=-x-1+Ce^x$

24. $y=\dfrac{ce^{bx}}{a+b}+Ce^{-ax}$

25. $y=1+Ce^{-\frac{x^3}{3}}$

26. $y=(1+x)^n\left(e^x+C\right)$

27. $y=x+Cx(x-1)$

28. $y=\ln x-1+\dfrac{C}{x}$

29. $y=\dfrac{x^5}{3}+Cx^2$

30. $y=\dfrac{x+C\sqrt{1+x^2}}{1+x^2}$

31. $y=e^{-\sin x}\left(x+C\right)$

32. $y=(C-2\cos x)\cos x$

33. $y=e^x+Cx$

34. $y=-\dfrac{a(\cos x+b\sin x)}{1+b^2}+Ce^{bx}$

35. $y=\dfrac{1}{3}\dfrac{x^3+3x+3C}{\left(x^2+1\right)^2}$

36. $\dfrac{1}{y^2}=-a\left(x^2+\dfrac{1}{2}\right)-Ce^{2x^2}$

37. $\dfrac{1}{y^2}=x+\dfrac{1}{2}+Ce^{2x}$

38. $\dfrac{1}{y}=\dfrac{x^2+C}{2x}$

39. $\sqrt{y}=x-2+Ce^{-\frac{x}{2}}$

40. $\dfrac{1}{y}=x+C\sqrt{x^2-1}$

41. $y^2=-ax\ln x+Cx$

42. $\dfrac{x}{y}-\dfrac{1}{2y^2}=C$

43. $\sqrt{y} = \tan x + \frac{1}{x}\ln\cos x + \frac{C}{x}$

44. $\frac{1}{y} = -\frac{x^5}{3} + Cx^2$.